

On the mechanism of the pulsed high energy emission from the pulsar PSR B1509-58

Chkheidze N.* Osmanov Z.

Centre for Theoretical Astrophysics, ITP, Ilia State University, 0162-Tbilisi, Georgia

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ABSTRACT

We investigate the high-energy (HE) ($< 1\text{GeV}$) emission from the pulsar PSR B1509-58 and its relation to the radio emission in the 1.4GHz frequency band. The role of the quasi-linear diffusion in producing the pulsed HE radiation is investigated. We show that by means of the cyclotron instability the relatively low frequency waves excite, which due to the diffusion process influence the particle distribution function and switch on the synchrotron emission mechanism. We argue that the coincidence of HE main peak and the radio pulse is a direct consequence of the fact that the high and low frequency radiation is produced simultaneously in a local area of the pulsar magnetosphere. In the paper we also consider the absence of the radio counter pulse and explain this fact.

Key words: Pulsars: individual: PSR B1509-58 – Radiation mechanisms: non-thermal.

1 INTRODUCTION

Recent development in the field of high energy γ -ray astronomy stimulates new theoretical approaches and methods in modern astrophysics. Last decade the bulk of observational results in the high and very high energy domains comes from several telescopes: High Energy Stereoscopic System (HESS), Very Energetic Radiation Imaging Telescope Array System (VERITAS), Major Atmospheric Gamma-ray Imaging Cherenkov (MAGIC) Telescope etc. These instruments have detected several sources at GeV and TeV energies (e.g. Mariotti 2010; Abramowski et al. 2011; Ong & Mariotti 2010) and Fermi Large Area Telescope (Fermi-LAT) in turn, announced the first catalogues of active galactic nuclei (AGN) (Abdo et al. 2010a) and pulsars (Abdo et al. 2011), respectively.

In the context of the present paper a special interest deserves the detection of the HE $< 1\text{GeV}$ γ -ray pulsed emission from the young and energetic pulsar PSR B1509-58 by the Fermi-LAT with a light curve presenting two symmetric peaks separated by phase 0.37 ± 0.02 (see Fig.1 from Abdo et al. (2010b)). The γ -ray signals are offset from the main radio peak observed at 1.4GHz by phases 0.96 ± 0.01 and 0.33 ± 0.02 . The first γ -ray peak is almost coincident with the radio pulse. Particularly, leading the main radio pulse by phase 0.04 ± 0.01 . The results of Fermi-LAT are obtained for the energy interval 30MeV-100GeV and it is

shown that up to 1GeV the emission reveals pulsations. We focus on explaining the pulsed HE γ -ray emission of this source in 1MeV-1GeV interval.

According to the observational evidence, the main pulses in the radio (1.4GHz) and HE ($< 1\text{GeV}$) bands are almost coincident, meaning that they are generated simultaneously, in a certain localized area of the pulsar magnetosphere. In this context the quasi-linear diffusion (QLD) could be an efficient mechanism providing the coincidence of pulses in the aforementioned domains. Kazbegi et al. (1992) showed that if the magnetic energy density exceeds that of plasmas the low frequency cyclotron modes are excited. In turn, the unstable cyclotron waves interact with resonant particles by means of the QLD and as a result they acquire the pitch angles, leading to the efficient synchrotron emission process as it was argued by Lominadze et al. (1979) and Machabeli & Usov (1979). For high Lorentz-factors the synchrotron emission comes in the HE domain. In a series of papers, the mechanism of QLD was applied to active galactic nuclei (Osmanov & Machabeli 2010; Osmanov 2010a,b, 2011) and pulsars (Malov & Machabeli 2001; Chkheidze & Machabeli 2007; Machabeli & Osmanov 2009, 2010; Chkheidze et al. 2011), respectively. Recently, we have applied this mechanism to explain the pulsed very high energy ($> 25\text{GeV}$) radiation of the Crab pulsar (Machabeli & Osmanov 2009, 2010; Chkheidze et al. 2011) and it's pulse-phase coincidence with the optical signals (Aliu et al. 2008).

* E-mail: nino.chkheidze@iliauni.edu.ge

In the present paper we implement the mechanism of the QLD to the pulsar PSR B1509-58. The observations confirm that the signals of the HE γ -rays ($< 1\text{GeV}$) are almost coincident in phase with the radio signals (1.4GHz). Thus, we argue that they are produced simultaneously in one location of the pulsar magnetosphere.

The paper is organized as follows. In Section 2 we consider our model of the quasi-linear diffusion and apply it to the pulsar PSR B1509-58, explaining major characteristics of emission; in Sect. 3 we discuss the model and in Sect. 4 we summarize our results.

2 MODEL

According to the standard model of pulsars, it is believed that the pulsar's magnetosphere is filled by electron-positron plasma, which consists of three components. The so-called primary beam particles, which uproot from the neutron star's surface and are described by the Goldreich-Julian density (Goldreich & Julian 1969) in due course of time accelerate in the gap electric field and reach a corresponding threshold when the cascade pair creation becomes inevitable (Sturrock 1971; Tademaru 1973). Therefore, the pulsar's magnetosphere is filled by the primary beam ($\gamma \sim \gamma_b$), the bulk of plasma ($\gamma \sim \gamma_p$) and the so-called tail particles ($\gamma \sim \gamma_t$) (see Fig1. from Arons (1981)). The distribution function of the electron-positron plasma is one-dimensional, as at the pulsar surface any transverse momenta of relativistic electrons are lost in a very short time ($\leq 10^{-20}\text{s}$) via synchrotron emission in very strong magnetic fields.

According to the work of Kazbegi et al. (1992), by means of the anomalous Doppler effect plasmas in the pulsar magnetosphere undergo the cyclotron instability. In particular, due to the curvature radius of the field lines, ρ , the beam particles are forced to drift perpendicularly to plane of the curved field line with the following velocity

$$u_x \equiv \frac{cV_{\parallel}\gamma_b}{\rho\omega_B} \quad (1)$$

where c is the speed of light, V_{\parallel} is particle velocity along the field line, $\omega_B \equiv eB/mc$ is the cyclotron frequency, B is the magnetic induction, e and m are the electron's charge and the rest mass, respectively. The distribution function is one-dimensional and anisotropic and plasma becomes unstable. Both of these factors (the one-dimensionality of the distribution function and the drift of particles) might cause generation of eigen modes in the electron-positron plasma if the following resonance condition is satisfied

$$\omega - k_{\parallel}V_{\parallel} - k_x u_x - \frac{\omega_B}{\gamma_b} = 0. \quad (2)$$

Here, k_{\parallel} is the longitudinal component of the wave vector and k_x is the wave vector's component along the drift. From Eq. (2) one can see that the pure transversal modes (t -waves) characterized by the following dispersion relation (Kazbegi et al. 1992).

$$\omega_t \approx kc(1 - \delta), \quad \delta = \frac{\omega_p^2}{4\omega_B^2\gamma_p^3}, \quad (3)$$

where k denotes the modulus of the wave vector, $\omega_p \equiv$

$\sqrt{4\pi n_p e^2/m}$ is the plasma frequency and n_p is the plasma density. In the framework of the present model, the aforementioned waves, by means of the QLD, influence the resonant particles leading to the diffusion of particles along and across the magnetic field lines. Consequently, particles acquire pitch angles and start to radiate in the synchrotron regime.

It should be mentioned that we study the process of the QLD close to the light cylinder surface, where, as it was investigated, effects of centrifugal acceleration are very efficient for pulsars (Osmanov & Rieger 2009) and AGN (Osmanov et al. 2007). Particularly, for explaining high and very high energy emission of pulsars one needs a mechanism providing very high energies of electrons, which is not provided by the gap acceleration process. As it was shown in Osmanov & Rieger (2009), the centrifugal acceleration process is quite efficient in the light cylinder zone and can provide the reacceleration of the primary beam particles to reach the Lorentz factors of the order of 10^7 . This in turn leads to the HE emission pattern, that is in a good agreement with the observational data (Osmanov & Rieger 2009).

In order to describe the process of the QLD, one has to take into account as the dissipative factors, which try to decrease the pitch angles, as the diffusion attempting to increase them. When relativistic electrons emit in the synchrotron regime, they undergo the so-called reaction force, having as longitudinal as transversal components (Landau & Lifshitz 1971)

$$F_{\perp} = -\alpha \frac{p_{\perp}}{p_{\parallel}} \left(1 + \frac{p_{\perp}^2}{m^2 c^2} \right), \quad F_{\parallel} = -\frac{\alpha}{m^2 c^2} p_{\perp}^2, \quad (4)$$

where $\alpha = 2e^2\omega_B^2/3c^2$, p_{\perp} and p_{\parallel} are transversal and longitudinal components of momentum, respectively.

On the other hand, the diffusion process, under certain conditions, might balance the corresponding dissipation, leading to the quasi-stationary state ($\partial/\partial t = 0$), which, as is shown by Chkheidze et al. (2011), for $\partial/\partial p_{\perp} \gg \partial/\partial p_{\parallel}$ results in the following kinetic equation governing the aforementioned mechanism

$$\frac{\partial}{\partial p_{\perp}} (p_{\perp} F_{\perp} f) = \frac{\partial}{\partial p_{\perp}} \left(p_{\perp} D_{\perp,\perp} \frac{\partial f}{\partial p_{\perp}} \right), \quad (5)$$

where f is the distribution function of electrons,

$$D_{\perp,\perp} \equiv \frac{e^2 |E_k|^2 \delta}{8c}, \quad (6)$$

is the diffusion coefficient and E_k is the electric field. In order to estimate the value of $|E_k|^2$, we assume that half of the beam energy density, $mc^2 n_b \gamma_b / 2$ (n_b is the beam density), converts to the energy density of the waves $|E_k|^2 k$. Therefore, an expression of $|E_k|^2$ writes as follows

$$|E_k|^2 = \frac{mc^2 n_b \gamma_b}{2\omega}. \quad (7)$$

The solution of Eq. (5) is given by

$$f(p_{\perp}) = C \exp \left(\int \frac{F_{\perp}}{D_{\perp,\perp}} dp_{\perp} \right) = C e^{-\left(\frac{p_{\perp}}{p_{\perp 0}} \right)^4}, \quad (8)$$

where

$$p_{\perp 0} = \left(\frac{4\gamma_b m^3 c^3 D_{\perp,\perp}}{\alpha} \right)^{1/4}, \quad (9)$$

and for the mean value of the pitch angle we obtain

$$\langle \psi \rangle = \frac{p_{\perp 0}}{2\gamma_b m c}. \quad (10)$$

2.1 Radio emission

According to the present model, the generation of the synchrotron emission is provided by the feedback of the low frequency cyclotron waves on the resonant particles. It is easy to show from Eqs. (2, 3) that the unstable cyclotron waves will have the following frequency (Machabeli & Usov 1979)

$$\nu \simeq \frac{e^2 B^2 P}{2\pi^2 m^2 c^2} \frac{\gamma_p^2}{\gamma_b^4} \times 10^{-9} \text{GHz}. \quad (11)$$

The estimations show that for reasonable magnetospheric parameters (we assumed that $\gamma_p \sim 1$ and $\gamma_b \sim 10^7$ (Osmanov & Rieger 2009)) the anomalous Doppler effect leads to the excitation of the radio waves (1.4GHz), which in turn influences the particles and switches on the synchrotron radiation process giving the HE γ -ray emission. According to Abdo et al. (2010b) it is shown that the main pulses of the HE ($< 1\text{GeV}$) γ -rays and the radio emission come with the same phases. We argue that this observation feature is caused by the fact that the emission in both domains are produced simultaneously and in one location of the magnetosphere - close to the light cylinder surface.

2.2 Spectrum of the high energy emission

In this subsection we are going to explain the observed spectrum of the HE γ -rays. Following the method developed by Ginzburg (1981), one can show that the flux of the synchrotron emission is given by the following expression (Chkheidze et al. 2011)

$$F_\epsilon \propto \int_{p_{\parallel \min}}^{p_{\parallel \max}} f_{\parallel}(p_{\parallel}) B \langle \psi \rangle \frac{\epsilon}{\epsilon_{GeV}} \left[\int_{\epsilon/\epsilon_{GeV}}^{\infty} K_{5/3}(z) dz \right] dp_{\parallel}, \quad (12)$$

where $f_{\parallel}(p_{\parallel})$ is the longitudinal distribution function of electrons. The major difference from the model of Ginzburg is that according to our scenario the synchrotron emission is generated in the outer part of the pulsar magnetosphere via the QLD, which inevitably restricts the values of the pitch angles. On the other hand, according to Ginzburg (1981), along the line of sight the magnetic field is chaotic leading to the broad interval of the latter $\psi \in (0; \pi)$. This difference is principle between the standard approach and the method presented in this paper. Therefore, in Eq. (12) instead of ψ we put it's average value (see Eq. (10)), which, unlike the standard model, is the dynamical parameter and participates in integration.

In order to find out the spectrum of the HE emission, one needs to know the behaviour of the distribution function, $f_{\parallel}(p_{\parallel})$. For solving this problem, we consider the equation governing the evolution of $f_{\parallel}(p_{\parallel})$ (Chkheidze et al. 2011)

$$\frac{\partial f_{\parallel}}{\partial t} = \frac{\partial}{\partial p_{\parallel}} \left(\frac{\alpha_s}{m^2 c^2 \pi^{1/2} p_{\perp 0}^2} f_{\parallel} \right). \quad (13)$$

We are interested in the stationary state ($\partial/\partial t = 0$), of the QLD, which reduces the aforementioned equation and leads to the following form of the distribution function

$$f_{\parallel} \propto \frac{1}{p_{\parallel}^{1/2} |E_k|}. \quad (14)$$

On the other hand, the cyclotron noise is described by the equation

$$\frac{\partial |E_k|^2}{\partial t} = 2\Gamma |E_k|^2 f_{\parallel}, \quad (15)$$

where the increment of the instability, Γ , is given by (Kazbegi et al. 1992)

$$\Gamma = \pi \frac{\gamma_b \omega_b^2 \delta}{\gamma_p \omega_B}. \quad (16)$$

Here $\omega_b \equiv \sqrt{4\pi n_b e^2/m}$ is the plasma frequency of the resonant (beam) electrons.

By taking into account Eqs. (15,16) one can reduce Eq. (13) to the following form (Chkheidze et al. 2011)

$$f_{\parallel} - \beta \frac{\partial}{\partial p_{\parallel}} \left(\frac{|E_k|}{p_{\parallel}^{1/2}} \right) = f_{\parallel 0}, \quad (17)$$

where

$$\beta = \left(\frac{4}{3} \frac{e^2}{\pi^5 c^5} \frac{\omega_B^6 \gamma_p^3}{\omega_p^2} \right)^{1/4}, \quad (18)$$

and $f_{\parallel 0}$ is the initial distribution function of particles. Since the electron number density behaves with distance as $1/r^3$, it is clear that one has the following condition, $f_{\parallel} \ll f_{\parallel 0}$, that reduces Eq. (17)

$$\beta \frac{\partial}{\partial p_{\parallel}} \left(\frac{|E_k|}{p_{\parallel}^{1/2}} \right) + f_{\parallel 0} = 0. \quad (19)$$

As we can see the function $E_k(p_{\parallel})$ drastically depends on the form of the initial distribution of the primary beam electrons. According to the work, (Goldreich & Julian 1969), a spinning magnetized neutron star generates an electric field which extracts electrons from the star's surface and accelerates them to form a low-density ($n_b = B/Pce$) and energetic primary beam, which in turn, is reaccelerated in the light cylinder zone and might reach quite high Lorentz factors, $\sim 10^7$, (Osmanov & Rieger 2009). We only know the scenario of creation of the primary beam, but nothing can be told about its distribution, which drastically depends on the neutron star's surface properties and temperature. To our knowledge there is no convincing theory which would predict the initial form of the distribution function of the beam electrons. Thus, we can only assume that the beam electrons have the power-law distribution, $f_{\parallel 0} \propto p_{\parallel}^{-n}$, and for $|E_k|^2$ we will obtain the following behaviour

$$|E_k|^2 \propto p_{\parallel}^{3-2n}. \quad (20)$$

In order to simplify Eq. (12), we replace the integration variable p_{\parallel} by $x \equiv \epsilon/\epsilon_{GeV} \simeq 3.6 \times 10^{18} \nu \gamma^{-9/4}$, and taking into account Eqs. (6,9,10,14,20) the expression of the flux will be

$$F_\epsilon \propto \epsilon^{-\frac{2-n}{4-n}} \int_{x_{\min}}^{x_{\max}} x^{\frac{2-n}{4-n}} \left[\int_x^{\infty} K_{5/3}(z) dz \right] dx. \quad (21)$$

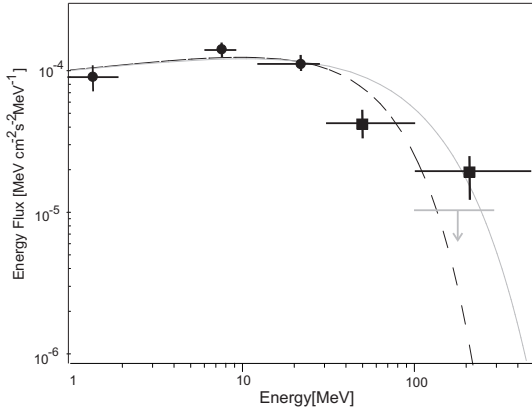


Figure 1. Spectral energy distribution of PSR B1509-58 (dashed-line corresponds to the model spectrum $\epsilon^{-1.87} \exp[-(\epsilon/0.059)^{1.3}]$ and the solid-line to $\epsilon^{-1.87} \exp[-(\epsilon/0.081)]$ obtained by Pilia et al. (2010) from a fit of pulsed fluxes from soft to hard γ -rays. The round points represent COMPTEL and square points AGILE observations (Kuiper et al. 1999; Pilia et al. 2010), respectively. The arrow represents Fermi upper limit (Abdo et al. 2010b).

As we see from this equation the HE spectral index is given by the following expression $(2 - n)/(4 - n)$. According to Pilia et al. (2010) the observed HE pulsed emission of PSR B1509-58 in the energy domain (0.001 – 1) GeV is best described by a power-law plus cutoff with the spectral index equal to 1.87. When $n = 6.3$ the spectral index of the synchrotron emission equals 1.87 (see Eq. (21)). If we assume that the energy of the emitting electrons vary between $\gamma_{min} \simeq 3 \cdot 10^6$ and $\gamma_{max} \simeq 3.5 \cdot 10^7$, one can show that the integral (21) can be approximately expressed by the following function

$$F_{\epsilon} \propto \epsilon^{-1.87} \exp \left[- \left(\frac{\epsilon}{0.059} \right)^{1.3} \right]. \quad (22)$$

As we can see our emission scenario predicts the exponential cutoff, with the cutoff energy 59 MeV (see Fig. 1).

3 DISCUSSION

It is clear that the present model provides the self consistent explanation of several interesting observational features of the pulsar PSR B1509-58. In particular, we have already seen that for reasonable magnetospheric parameters the QLD can support the HE pulsed emission up to 1 GeV. According to the observations, as it was shown by Abdo et al. (2010a), the main pulses of the HE domain are almost coincident with those of the radio emission on the frequency 1.4 GHz. In the framework of our model this might mean that radiation in both domains are produced almost simultaneously and in one location of the pulsar magnetosphere. We have shown that by means of the cyclotron instability, the radio waves are excited, which influence the resonant particles and cause their diffusion as along also across the magnetic field lines. Consequently, they acquire the pitch angles and the synchrotron emission process is switched on. This is the reason, why the HE γ -rays and the low frequency waves are strongly connected. On the other hand we neglect the role of the curvature radiation, because

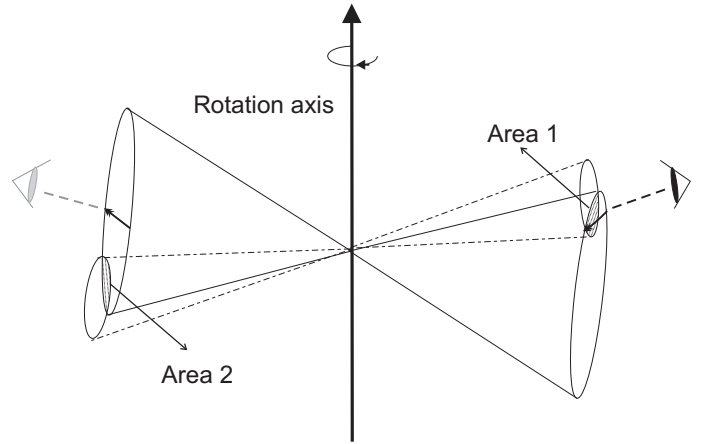


Figure 2. Emission geometry of PSR B1509-58.

otherwise the main pulse of the HE emission would be wider than it is observed. This automatically means that the curvature radius of the magnetic field lines are large enough to insure the negligible role of the curvature radiation process. As it was shown by Osmanov et al. (2008, 2009) such a configuration of the field lines might be provided by means of the so-called curvature drift instability, that makes the magnetic field lines rectify efficiently in the very vicinity of the light cylinder surface. In particular, as it was shown in the aforementioned articles, the curvature drift (see Eq. (1)) will create the corresponding current, leading to the creation of additional magnetic field. On the other hand, this toroidal field, by means of the centrifugal effects, will efficiently amplify, changing the overall configuration of the magnetosphere. It has been found that the curvature drift instability has two general modes, one of which makes the field lines more curved and the other makes them rectify. We assume that since the HE profile of the main signal is relatively narrow, the curvature radiation should be negligible and therefore, the field lines must be almost straight, which is provided by the curvature drift instability.

In the framework of the present paper we provide theoretical confirmation of the measured HE spectrum of PSR B1509-58 in the energy domain (0.001 – 1) GeV. In particular, our model predicts the power-law plus an exponential cutoff, which provides an explanation of the observed decrease of the flux above 10 MeV (Abdo et al. 2010b). If the Lorentz factor of the resonant beam electrons are $\gamma_b \simeq 3 \cdot 10^6 - 3.5 \cdot 10^7$, then our theoretical spectrum $F_{\epsilon} \propto \epsilon^{-1.87} \exp[-(\epsilon/0.059)^{1.3}]$. As we can see the model automatically yields the exponential cutoff, with the cutoff energy 59 MeV.

Another important feature of the high energy γ -rays is that unlike the radio emission, apart from the main pulse it also has the second pulse, separated from it by the phase $\Delta\phi = 0.37 \pm 0.02$.

The observationally evident fact that the HE emission reveals two peaks might be a direct result of an emission geometry. In particular, we detect the pulsar emission twice per one rotation if the HE emission cone is almost perpendicularly inclined with respect to the rotation axis. According to the observations the main γ -ray pulse leads the radio pulse by phase 0.04, which indicates that the emission cones

of the HE and radio domains are not spacially coincident (see Fig. 2). In Fig. 2 cone 1 represents the HE emission cone, that has a common area (area 1 and area 2) with the radio emission cone (cone 2). Then it is clear that if the emission comes from area 1 which is parallel to the line of sight, we will see the emission profiles for the radio and HE bands, respectively. Due to star rotation and the corresponding emission channels, the counter HE radiation cone will have a position convenient for detecting the counter signal, whereas the counter radio cone will be significantly shifted, and the line of sight will be out of it. Therefore, as a result, we see the signal and the counter signal of HE emission and do not see the counter radio peak.

This approach explains another feature of the HE emission as well. As it is evident from the observations, the counter peak is wider with respect to the main pulse. In particular, from Fig. 2. it is clear that the time during which the line of sight stays inside the cone 1 is shorter than the time scale it stays inside counter HE cone.

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